Dance Battle is PSPACE-complete

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In a game of (rooted) Undirected Edge Geography (UEG) on a graph G, two players take turns extending a walk from an initial vertex v by an edge that does not already appear in the walk. The first player unable to do so loses. Dance Battle [2] is essentially a special case of UEG where G is the complement of a walk with no repeated edges.[1] UEG is known to be PSPACE-complete [1], and in this note, we show how to reduce an arbitrary instance of UEG to an instance of Dance Battle.

Let (G_0, v) be the input UEG instance. As the first step of the reduction, we transform G_0 into an equivalent graph G_1 where all vertices have odd degree. This is accomplished by attaching a three-vertex gadget to all vertices of even degree. Any player taking the edge into the gadget loses, so the gadgets do not affect optimal play. We output $G_2 = G_1 \oplus G_1$, the graph consisting of a disjoint union of two copies of G_1. Since the two copies are not connected, G_2 has the same value as G_1 and G_0.

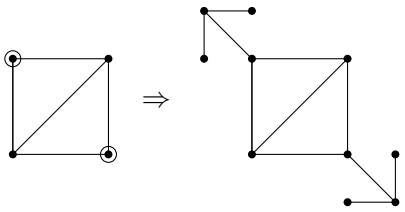

Figure 1: Adding gadgets to vertices of even degree.

By construction, G_2 has an even number of nodes, all of odd degree. The complement graph $\overline{G_2}$ is connected and has no nodes of odd degree. It follows that $\overline{G_2}$ has an Euler tour (including all self-loops in the original version of Dance Battle) and thus that G_2 corresponds to an instance of Dance Battle.

References

[1] Aviezri S. Fraenkel, Edward R. Scheinerman, and Daniel Ullman. Undirected edge geography. *Theor. Comput. Sci.*, 112(2):371–381, 1993.

[2] Puzzle Master. Dance Battle. `http://www.facebook.com/careers/puzzles.php?puzzle_id=12`.

[1] Dance Battle has self-loops, which do not complicate the reduction below.